Answer the following questions. Each question counts 10 points.

Calculators are not allowed

1) a) Find $f'(x)$ for the following functions:-

    (i) $f(x) = x \cot(x^2) + \sqrt{2x^3 + 1}$ (ii) $f(x) = \sin(\cos x) + \dfrac{\sec(3x)}{2x - 1}$

   b) Evaluate the following integrals:-

    (i) $\displaystyle\int \left(1 + \frac{1}{x}\right)^{-7} \frac{1}{x^2}\, dx$ (ii) $\displaystyle\int \frac{\cos(4x)}{\sqrt{1 - \sin(4x)}}\, dx$ (ii) $\displaystyle\int_{1}^{2} x\sqrt{1 + x^2}\, dx$

2) a) Evaluate the following limits if exist:-

    (ii) $\displaystyle\lim_{x \to 0}\left(\frac{1 - \cos(2x)}{x} + \frac{x}{\sin(3x)}\right)$ (ii) $\displaystyle\lim_{x \to \infty}\left(\sqrt{x + 9} - \sqrt{x}\right)$

   b) Use differentials to approximate the value of $\sqrt[3]{26.9}$

   c) Given $f(x) = \displaystyle\int_{0}^{x}(t^2 + 1)\,dt$, for $x > 0$. Show that $f(x)$ is an increasing function for all x> 0.

3) a) If $f(x) = x^5 + 9x + 1$, Show that $f(x)$ is an increasing function and the equation $f(x) = 0$ has exactly one real root.

   b) Given $I = \displaystyle\int_{-6}^{3}\sqrt{x + 6}\, dx$, Find the number z that satisfies the conclusion of the Mean Value Theorem for definite integrals

4) a) Find the equation of the normal line to the graph

    $\cos(x + y) + x^2 y^2 - x = 0$,   at the point $(0, \frac{\pi}{2})$.

   b) Find the dimensions of a rectangle of maximum area that has perimeter 20 cm.

5) Let $f(x) = \dfrac{1}{x - x^2}$

   a) Find the domain of f

   b) Find the vertical and horizontal asymptotes, if any.

   c) Find the intervals where f is increasing or decreasing.

   d) Find the local extrema of f.

   e) Discuss the concavity of f

   f) Sketch the graph of the function.

Good luck